Robust Optimization

This version: (First official draft)

Course instructors

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Further instructors will be asked to join the course, and may do so based on availability and interest

Language: English

Application procedure

Goal and target audience

The seminar requires solid knowledge in advanced mathematics, especially the knowledge of linear algebra, probabilities and fundamental optimization, etc. Mathematical maturity and the ability to write down precise and rigorous arguments and proofs are also important. Computer programming skills are expected. Doctoral students are the target audience.

Application process

Self-registration in Moodle.

Course aims

What this course is

This is a PhD course on robust optimization. The decision-making problem requires the parameters of underlying models. While applying data to estimate parameters, the uncertainty is unavoidable. Robust optimization is an emerging area to incorporate uncertainty into mathematical programming models. This course will cover various aspects of robust optimization and satisficing frameworks, including non-stochastic and stochastic models.

What this course is not

The course is not covering basic data analytics, probability theory and optimization.

Course objectives

Knowledge Objectives

After taking this course, participants shall be able to understand the theoretical foundations of robust optimization.

Skills Objectives

Participants shall learn the robust modeling techniques, the ways to build the uncertainty (ambiguity) set based on data and the robust counterpart approach to convert the problem into a solvable equivalent.

Learning Objectives
Participants will understand where the uncertainty of parameters comes from and the impact of uncertain parameters on downstream decisions. They shall be able to apply the idea of robust modeling in their research fields.

Preliminary schedule

Kick-off meeting via Zoom by annoucement. The course will be taught as a series of seminars on Thursday morning from 9:00 am to 12:00 am weekly. Dates could be coordinated with participants. If possible, the course will be held in person; otherwise, via Zoom.

Core readings

- Boyd, S., Boyd, S. P., & Vandenberghe, L. (2004). *Convex optimization*. Cambridge university press.
- Bertsimas, D., & Tsitsiklis, J. N. (1997). *Introduction to linear optimization* (Vol. 6, pp. 479-530). Belmont, MA: Athena Scientific.
- Ben-Tal, A., El Ghaoui, L., & Nemirovski, A. (2009). Robust optimization. In *Robust optimization*. Princeton university press.

Course procedures

Our course is a mix of seminar presentations by the participants and group discussions. There will be an intense 180-minutes session presentation and discussion weekly. During this time, we are going to discuss a specific topic in RO and analytics in greatest depth. The following topics will be covered:

1: Classical Robust Linear Optimization
- Bertsimas, Dimitris, and Melvyn Sim. "The price of robustness." *Operations research* 52.1 (2004): 35-53.
- Satisficing Perspective of RO
- Min-Max Linear Problems
- Infimum Convolution & Norms
- Robust Optimization Duality

2: Distributionally Robust Optimization
- Decision under Risk
- Tractable Ambiguity Sets
    - Continuous distribution with support and moment constraints

3: Event-wise Ambiguity Set
- Mixture distribution
- Wasserstein distance
- Partitioned based methods
    - Tree based
    - K-means

4: Monetary Risk Measures
- Convex and Coherent Risk Measures
- Worst-case VaR and CVaR

5: Satisficing
- Satisficing Measures
- Robust Satisficing

6: Stochastic Programming

- Two stage problem
- Multiple stage problem

7: Adaptive Robust Optimization
- Static Recourse Adaption
    - Fixed Recourse
    - Complete Recourse
- Affine Recourse Adaptation
- Strengthening Recourse Formulation
- Dual Recourse Adaptation
- Multi-period ARA

8: Adaptive Distributionally Robust Optimization

9: Adaptive Robust Stochastic Optimization

10: Convex Analysis
- Basic Convex Analysis
- KKT Conditions
- Conic Optimization
- Second Order Cone Optimization Problem (SOCP)
- Semidefinite Optimization Problem (SDP)

11: Convex Analysis and Robust Modeling
- Minmax Saddle Function
- Tractability Robust Counterpart
- Robust Quadratic Optimization
- Robust SOCP
- Approximate Robust Counterpart
- Entropic Methods
    - Probability Inequality: CVaR Approximation
    - Expected Excess
    - Entropic Deviations
    - Adaptive Distributionally Robust Optimization
    - Robust Combinatorial Optimization

Assessment

Participants will be assessed based on their attendance and presentation (50%), and assignments (50%). The course is pass/fail, not graded. In order to pass the course, participants should take part in all classes. In case of excused absence due to illness they need to hand in a written assignment about the content of the class they have missed.

Each participant will present at least one topic and lead the discussion. Everyone else should read the relevant literature before class and participate actively in the discussion. Students who show up unprepared and/or unable to contribute actively may be considered absent.

Workload

6 ECTS (48 contact hours, 180 hours total workload)